Deviance and Vice: strength as a theoretical virtue in the epistemology of logic

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In recent years there has been renewed interest in a broadly abductive approach to the epistemology of logic. (Priest, 2006; Russell, 2014; Hjortland, 2017; Beall, 2017; Williamson, 201X) While the details vary, the core idea is that rival logics are different theories of the relation of logical consequence, and the theory we should accept is the one which is adequate to the data and which surpasses other data-adequate theories in theoretical virtue—e.g. simplicity, strength, elegance, unity, symmetry, or ontological parsimony—while harbouring as little as possible theoretical vice—such as ad hockery, inelegance, or ontological profligacy.

As (Hjortland, 2017, 632) has noted, agreement on a general epistemic approach has not resulted in similar agreement about which is the correct logical theory. Priest holds that the correct logic is paraconsistent, Williamson (201X) that it should be classical—higher-order modal classical logic according to (Williamson, 2013))—Beall (2017) argues that the one true logic is the Logic of First Degree Entailment (FDE), and Hjortland and I are are pluralists—though of different kinds—a view which might prima facie seem at odds with the idea that the epistemology of logic works by inference to *the* best exanplanation.

Looking further back in time, Quine endorsed a methodology of this kind, concluding that first-order classical logic was correct but that second-order and modal extensions were illegitimate.[1] Carnap too thought that finding the best logic was a matter of finding the best overall linguistic framework—something that would be judged by characteristics like usefulness and simplicity—but unlike Quine he embraced the prospect of non-classical logics: "The first attempts to cast the ship of logic off from the *terra firma* of the classical forms were certainly bold ones, considered from the historical point of view. But they were hampered by the striving after 'correctness'. Now, however, the impediment

[1] Still, Quine was not averse to the idea that classical might be given up in response to new data, such as that from quantum mechanics. (Quine, 1986, 85–86). However he seems to have regarded the costs of such a move as very high.

has been overcome, and before us lies the boundless ocean of unlimited possibilities."[2]

The broad agreement on methodology between rival research programs allows us to hope that we might eventually be able to determine which is the right logic to everyone's satisfaction. The widespread disagreement about the results among the same parties suggests that a description like "theory which is adequate to the data and exhibits most virtues and fewest vices" is not yet in very clear focus. If Beall can maintain that FDE satisfies it uniquely, and Williamson that classical logic does—even while they both agree on the content of those two logical theories—it suggests a lack of agreement about what the vices and virtues are, and likely also what data adequacy looks like.

There is no hope that the present paper will provide an algorithm for choosing the best logic. But there is a *lot* of space between where we are now, and such an algorithm. My broad aim with the project of which the present paper is a part is to get clearer on some of the details of the abductive method in logic with the aim of making the choice of logics easier. The present paper has a narrower aim—in the interests of having a thesis for people to get their teeth into at the Rutgers Epistemology Conference—and focuses on the theoretical characteristic of strength. Strength has often been thought to be a virtue in scientific theories and it is also often thought that *logical strength*—the well-defined relation between logics on which one is stronger than another if it contains all the theorems of the former but not vice versa—is a virtue in a logic.

> "Once we assess logics abductively, it is obvious that classical logic has a head start on its rivals, none of which can match its combination of simplicity and strength. Its strength is particularly clear in propositional logic, since PC is Post-complete, in the sense that the only consequence relation properly extending the classical one is trivial (everything follows from anything). First-order classical logic is not Post-complete, but is still significantly stronger than its rivals, at least in the looser scientific sense, as well as being simpler than they are; likewise for natural extensions of it to more expressive languages. In many cases, it is unclear what abductive gains are supposed to compensate us for the loss of strength involved in the proposed restriction of classical logic." (Williamson, 201X, p.19)[3]

Some supporters of sub-classical logics have agreed that logical strength is a virtue, even if it is one they are regretfully obliged to forgo for some other benefit, e.g.:

> "Weakening classical logic [...] is not something to be done lightly. There are sone obvious advantages to keeping classical logic even for "circular" predicates: advantages of simplicity, familiarity,

[2] (Carnap, 1937, xv) In his famous slogan: "In logic there are no morals." (Carnap, 1937, p.10 §17) Haack (1978) might also be thought to displays sympathies with abductivism.

[3] The reference to Post-completeness here makes it clear that *logical* strength is one of the flavours of virtuous strength that Williamson has in mind.

> and so on. Choosing to forgo these advantages has its costs. But I will argue [...] that the *disadvantages* of keeping classical logic for "circular" predicates are also very great, so that the undoubted cost of weakening the logic is worth bearing." (Field, 2008, 15)

This concessive view is common and it is compatible with the idea that logical strength is a genuine virtue. A more radical view—sometimes hinted at recently in e.g. (Hjortland, 2017) (Shapiro, 2014, Ch3) and (Beall, 2017)—would be that logical strength is really a theoretical *vice*, and equivalently, that logical *weakness* is a theoretical virtue, perhaps because weaker logics allow for the development of a broader or better range of extra-logical theories, or allow us to distinguish more possibilities (e.g. the possibility that $\neg\neg P$ is true from the possibility that P is true.) Summing up such radicals, Williamson writes:

> "One often encounters various forms of exceptionalism about logic, according to which weakness is a strength in logic, because weak logics leave open more possibilities, prejudge fewer issues, and achieve higher levels of neutrality." (Williamson, 201X, 18)

Contrary to all these views, I will argue in this paper that logical strength is neither a theoretical virtue nor a theoretical vice in a logical theory. If a characteristic is a theoretical virtue, I will assume, then a theory which has more of it is always—all else being equal—better than a theory which has less. But neither logical strength nor logical weakness are like this. Logical theories are supposed to get the extension of the entailment relation *right*. If they overshoot (include too many arguments in the extension of the relation) then they will be too strong. If they undershoot (include too few) then they will be too weak. So more strength would be an improvement for logics that are too weak, but a degeneration in logics that are already strong enough. And more weakness would be an improvement for logics that are too strong, but a degeneration in logics that are already weak enough. The right amount of logical strength is not "as strong as we can manage" but rather a Goldilocks level: not too weak, not too strong.

However, there is a different kind of strength, less easily defined, which is more a matter of informativeness and precision in a theory. Williamson calls this "scientific strength" and here I will agree with him that scientific strength is a virtue in logical theories. But I will also argue that classical and subclassical logics like LP (Williamson's target) are on a par with respect to scientific strength. If this is right then I need to respond to Williamson's claim that logical strength entails scientific strength.

The first section of the paper introduces the general abductive approach and distinguishes it from anti-exceptionalism and Quineanism more generally. The second section looks at logical strength and argues that it is neither a theoretical virtue nor a theoretical vice. The third section looks at scientific strength and argues that a particular set of logics which differ in logical strength are on a par with respect to scientific strength.

1 Abductivism and what abductivism is not

The heart of the abductivist approach consists in two claims: first, it is holist about the justification of logic; it is entire theories—rather than isolated claims—which are accepted or rejected. Second, the accepting or rejecting is done on the basis of adequacy to the data, and then the possession of theoretical virtues and vices.

The abductive approach has historically been associated with a more broadly anti-exceptionalist view of logic (Quine, 1951, 1986). Exceptionalism in the philosophy of logic is the view that logic has a series of characteristics that make it unlike empirical sciences, such as physics. For example, while physics is usually thought to be a posteriori, synthetic, descriptive, empirical, contingent, factual, "about the world", and not-metalinguistic, logic has sometimes been thought to be a priori, analytic, necessary, normative, intuitive, basic, "obvious", about language or perhaps topic neutral, not "about the world", argumentatively neutral in debates, and metalinguistic. Anti-exceptionalists about logic deny—to varying degrees—that logic has one or more of features that make it different from the empirical sciences. For example, Harman (1986) argues that logic is not normative, Williamson that it is not epistemically analytic (Williamson, 2008, Ch4) and not metalinguistic (Williamson, 2013, p.93), Hjortland (2017) that it is not a priori (though it is metalinguistic), and most famously, Quine, in the last part of "Two Dogmas", argued that the epistemology of of logic was broadly abductive and that as a result, it was neither analytic, nor apriori, nor necessary.[4]

Because of this history, it's important for me to stress that one may be an abductivist without being terribly Quinean about logic in other respects—that is, abductivism is *independent* of many of these other issues. For example, one may hold that the epistemology of logic is abductive, without thinking that logic is a posteriori. If one thought that the evidence for a logic is itself a priori (perhaps defeasible intuitions about the validity of particular interpreted arguments) and that the vices and virtues on which a theory should be judged are also discoverable apriori—perhaps elegance and simplicity are such things?—then one could be both an abductivist and an apriorist about logic.

On the other hand, if one thought that logical theories should be assessed alongside physical theories, and the resulting pairings assessed for empirical adequacy and virtues (e.g. quantum mechanics plus classical logic, vs quantum mechanics plus quantum logic) then a posteriori data is relevant to the data-adequacy of logics. Or alternatively, if one thinks that logical theories stand or fall on their own, but that an important theoretical virtue is *usefulness when studying physics*, then again, one would be an abductivist who thinks that logic is a posteriori.

Perhaps most importantly for compatibility with contemporary work in logic, one can be an abductivist without holding that logic is contingent. Abductivism will mean that what appears to be the best logical theory at one time could later

[4]Of course, anti-exceptionalism comes in degrees, and one might deny that logic is exceptional in one way, while agreeing that it is exceptional in others.

be superseded by another (perhaps in response to new data-discoveries, perhaps in response to the development of a new, more virtuous theory.) That is, for abductivists, logic is rationally *revisable.* At the end of "Two Dogmas" Quine's working definition of analyticity was "statement that is true come what may", i.e. something which he assumed meant the statement was unrevisable. He and many of his readers also thought that analyticity was the best chance we had of accounting for necessity. So if "analytic" meant true come what may, and logic turned out to be revisable (and so *not* true come what may), then logic could not be analytic, and hence had little hope of being necessary.

Today, much has changed. Many philosophers think that statements can express necessary truths without being analytic. Elsewhere I have argued that analyticity concerns truth in virtue of meaning, which is a property which depends on meaning, and that the epistemology of meaning is also abductivist—so that theories of meaning, and their attendant results for analyticity, are themselves rationally revisable. Moreover, independently of my view of analyticity, we can recognise that the epistemology of necessary truths can allow that we rationally revise our beliefs about which claims are true. For example, *Hesperus is not Mars* is an a posteriori necessary truth whose justification depends on the place of Mars and Venus in our current theory of the solar system. An epistemically isolated community just beginning to study the skies might initially adopt a theory according to which Hesperus is not Mars, later conjecture that Hesperus might actually be Mars in response to a new theory about why Hesperus might appear red under certain conditions, confirm this theory with (somewhat misleading) data, and eventually give it up again once it is realised that the two are sometimes visible simultaneously. But this rational revisability of claims about the identity of Hesperus and Mars is compatible with the fact that *Hesperus is not Mars* is necessary. An abductive methodology in a certain domain and the attendant rational revisability of the claims we make about the domain are compatible with some or more of the truths of the domain being necessary.[5]

If abductively justified truths can be necessary, apriori and even true in virtue of meaning, one might begin wonder what abductivism actually rules out. One view that is incompatible with abductivism would be a one on which individual claims about entailment are justified atomistically, rather than in the context of a whole theory. Let an E-sentence be an atomic sentence in which '$\models$' is the main predicate[6], as it is in these two:

[5] A distracting complication: by the necessity of logic I mean the fact that the instances of many logical truths must express necessary truths (such as "Snow is white or it is not the case that snow is white") and also the fact that if a set of sentences Γ entails a sentence A then there is no possible world where every member of Γ is true and A false. This necessity is assumed to be a feature of many logics, and this is brought out when we add $\Box$ to the language—representing metaphysical necessity—and endorse the familiar principle of *necessitation*—a valid rule in all normal modal logics (including, say S4 and S5)—which allows us to 'Box' any logical truth: $\models A \Rightarrow \models \Box A$. That said, there are some important and interesting logics some of whose logical truths are contingent, e.g. in Kaplan (1989). See also Russell (2012) for further discussion.

[6] the 'E' is for entailment. I was writing '$\models$-sentence' until I realised that this is hard to

$$\begin{array}{rcl} A, A \to B & \vDash & B \\ & \vDash & \neg(A \wedge \neg A) \end{array}$$

And let an E-literal be either an E-sentence or its negation, as in:

$$\begin{array}{rcl} A \to B, B & \nvDash & A \\ & \nvDash & A \end{array}$$

Then epistemic atomism about logic holds that E-literals are justified E-literal-by-E-literal. This might be endorsed by someone who held e.g. i) that E-literals are justified by proving them from antecedently established E-literals, and that the most basic E-literals are a priori intuited to be correct or ii) that E-literals are established on the basis of model-theoretic proof or counterexample, and that it is obvious which model theory is the best, or iii) that E-sentences are established by formal, natural deduction proofs, whose basic rules are valid-in-virtue-of-meaning and easily recognisable as such by anyone who speaks the language.

What is distinctive about such an epistemically atomist view is that individual sentences are established or rejected on their own merits. And if this is right then the best theory could just be the set of all the individual sentences which have been established (and perhaps the negation of the disjunction of all the ones which have been refuted.) This would not be an abductivist view.

Still, I should note that even an abductivist can think that there is a place for atomistic proofs of E-sentences and counterexamples to not-E-sentences: *once we have chosen our logic*, we will be able to use (or at worst develop) a model theory and a proof theory for that logic, and these can then be used as usual to give proofs of individual E-sentences, and counterexamples to not-E-sentences.

Logical theories have two kinds of characteristic that might qualify as virtues or vices. The first is precise, well-defined and uncontroversial. This group includes completeness, compactness, decidability, and expressive power. The problem with this kind of characteristic is that it is frequently disputed whether it is a significant virtue (or vice). The other sort of characteristic is harder to pin down: simplicity, inelegance, symmetry, unity, ad hockery, etc. Here the status of the characteristic as a virtue or vice is rarely at issue—simplicity is virtuous, inelegance vicious—but the presence of the virtue in any one theory is disputed. So one reason strength seems like a promising characteristic to explore is that it seems to have elements of the attractive aspect of both groups of virtue: strength seems like simplicity, in that one might expect it to be uncontroversially a virtue, but like completeness, in that logics fall into well-defined strength hierarchies.

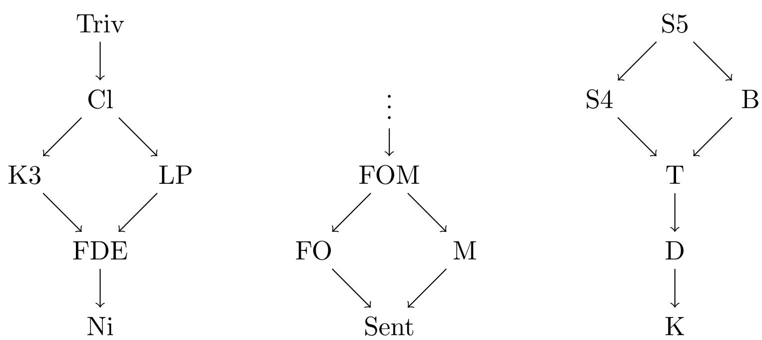
2 Logical Strength

Logicians are used to talking about a certain kind of strength. Limiting ourselves to sentential logics for a moment, classical logic is stronger than LP and stronger

read aloud.

than K3. Both LP and K3 are stronger than FDE. FDE is stronger than the empty logic—call that NL for the nihilist logic. And at the other end, the fact that classical logic is Post-complete means that the only logic strictly stronger than it is Triv—the trivial or universal logic on which every sentence A follows from every set of premises Γ.[7]

Expanding our set of logical constants produces another strength hierarchy; first-order logic is stronger than sentential logic, second-order logics are stronger than first-order logics, sentential modal logics are stronger than non-modal sentential logic. Of course, sentential modal logics form their own well-known hierarchy as well.



In each of these three diagrams, the arrows represent the same relation, 'is stronger than' and I will call this kind of strength *logical*:[8]

Definition 1 (Logical Strength (logics)) *Logic F is stronger than logic G just in case for all sets of sentences, Γ, and sentences, A,*

$$\textit{if } \Gamma \vDash_G A \textit{ then } \Gamma \vDash_F A$$

but not vice versa (that is, there is some Γ, A such that $\Gamma \vDash_F A$ but $\Gamma \nvDash_G A$.)

This is a special case of a more general notion of logical strength:

Definition 2 (Logical Strength (theories)) *Theory F is stronger than theory G just in case every sentence in theory F is in theory G, but not vice versa.*

We get the first definition from the second by thinking of logics as sets of sentences of the form $\Gamma \vDash A$ defined on languages.[9]

[7] There *are* non-trivial logics which have entailments that classical logic does not, but these must always lack some other entailment that classical logic is committed to; connexive logics, for example, may contain the non-classical logical truth $\vDash (A \to B) \to (\neg A \to \neg B)$ but they all lack the classical $\vDash \neg A \to [(A \to B) \to (A \to \neg B)]$. (Priest, 2006, p. 156)(Wansing, 2016)

[8] This is Williamson's terminology from Williamson (2013). Hjortland (2017) uses "deductive strength" to talk about the same relation.

[9] Some logicians are likely to want a more general conception of a logic still, perhaps allowing multiple conclusions and otherwise more complex sentences than these. Most of the logics mentioned in the present paper, however, are well-characterised by the set of sentences of the form $\Gamma \vDash A$ they license relative to a language, so I will stick with this characterisation for simplicity, with the caveat that things might get more complicated e.g. when we are talking about substructural logics. Section 3 of Hjortland (2017) is helpful on this topic.

Following Eklund (2012) we might call the structure that contains LP and K3 above a *horizontal* hierarchy (since the logics compete at the same (sentential) level) and the structure that contains first-order and modal logics a *vertical* hierarchy (in some sense, first-order logic is aimed at a level above that of sentential logic.) Still, it is logical strength that is represented by the arrows in both.

The modal logic hierarchy *might* be interpreted as a horizontal one—that is, one might think of the different modal logics as all offering different theories of the same logical constant, informally read as metaphysical necessity (say). But it's also quite common to think of $S4$ as giving a theory of a different symbol than $S5$ does; $\Box_{S4}$, rather than $\Box_{S5}$. If that is the case then the third diagram does not represent a horizontal hierarchy—the relationship between $S4$ and $S5$ would be more like that between first-order logic and sentential modal logic—and it would make sense to consider developing a hybrid modal logic combinging $\Box_{S4}$ and $\Box_{S5}$ (and perhaps even $\Box_D$ and $\Box_K$ etc.) much as we do first-order modal logic.

Anyway, though there are issues about interpreting these diagrams, let's press on. *If* logical strength were a theoretical virtue—or if it were a theoretical vice—then these rankings could be useful in choosing the correct logic. It is often thought that strength is a virtue in scientific theories, and it is sometimes thought that *logical* strength is a virtue in logical theories. Perhaps even more often, it is taken to be obvious that too much weakness would be bad. Non-classical sentential logics always seem to be in danger of being too weak. Too weak for what? Too weak to do metatheory, or too weak for closing theories. Too weak for mathematical proof, too weak to give a foundations for arithmetic or too weak for metaphysics. One might worry that if the correct logic is very weak, then logic might be too weak to be useful, even that logic as a discipline might not be very interesting. In any case, defenders of logics that have been weakened in one way (perhaps losing explosion) have often been quite keen to prevent them from being weakened in some other way (e.g. losing the law of excluded middle).

Recently, however, some logicians have begun to suggest an alternative view of logical strength, suggesting that weaker logics can be more valuable than stronger logics because they allow for the development and study of a greater variety of theories, such as theories of truth, arithmetic, or set theory.[10] Suppose, for example, we accept a logic as strong as classical logic. Then, on pain of triviality, paradox obliges us to reject a theory of truth which includes commitment to every instance of the disquotational schema:

$$True(\langle A \rangle) \text{ if and only if } A$$

Weaker logics can prevent the derivation of the contradiction, or prevent trivialisation once it is derived.

There are really two versions of the *pro-weakness* view: one which holds that strength is a virtue, but that we're forced to give it up in order to embrace

[10](Shapiro, 2014, Ch.3), (Beall, 2017) Hjortland (2017)

our best extra-logical theories and/or distinctions, and a more radical position which says that strength is a vice and weakness a virtue, because *quite generally* being able to entertain a large number of theories is beneficial to scientific discovery.[11,12]

So we have three views: on the first, logical strength is a virtue, on the second, it remains a virtue but is outweighed by other considerations. On the third, logical strength is a vice. Here is an argument that all three of those views are wrong, because logical strength is neither a virtue nor a vice in a logical theory. If a characteristic is a logical virtue (vice) in an area, then all things being equal, if theory F has more of that characteristic than theory G, then theory F is better (worse) than theory G. Logical strength is not like this.

Our strongest logic is Triv, the logic on which any arbitrary conclusion follows from any set of premises at all. Triv, it is widely agreed, is not our best logic. The most obvious problems with Triv are i) that it says that arguments are valid when they are not, and ii) that it is too strong.

Triv says that arguments are valid when they are not. For example, that *Snow is white* entails grass is purple, which is wrong. Triv is too strong, but what it is for a logic to be too strong is for it to say that at least one argument is valid when it is not. So the two big flaws with Triv are really just a single flaw: too much strength.

If a characteristic X is a theoretical virtue, there is really no such thing as too much X. The only way we would reject a stronger logic for a weaker one would be if that weaker logic had some *other* virtue, different from X, which counted for so much that its presence outweighed the value of the additional X. But I contend that the reason classical logic is obviously a much better logic than Triv is precisely that lacks some of the entailment commitments of Triv—i.e. it is better than Triv *because* it is weaker and *because* it doesn't commit to the false things about consequence that Triv does.

If this is right, then it is not the case that if a logic F is logically stronger than a logic G, then logic F is better than logic G. So logical strength is not a virtue. Parallel arguments (e.g. with Ni and LP) will show that logical strength is not a weakness either. Instead, the best level of logical strength is a matter of hitting a target: not too strong, not too weak. We can characterise that target precisely, and uncontroversially, though not in a way that provides a neutral arbiter between rival logics: a logic F has the correct level of strength iff for any set of sentences, Γ, and sentence A, $\Gamma \vDash_F A$ if and only if $\Gamma \vDash A$. *Snow*

[11] We might call this more radical view the "Sklavenmoral" approach to logical strength.

[12] Beall's view seems to have a bit of both. The weakness is motivated by a desire to handle paradoxes adequately, but he also—more radically—likes to rebrand weakness as the much more attractive sounding "depth": "Logic, on this 'deeper' picture, still affords a natural treatment of the paradoxes. The 'solutions' afforded by standard (though lopsided) subclassical logics carry over to FDE. The logic is weak (or 'deep') enough to accommodate standard paradoxical notions (e.g., truth, exemplification, etc.) By diving deeper than the standard lopsided subclassical levels we do not lose the options for naturally resolving paradoxes; we have more options—treating some of them as 'gappy phenomena' and some 'glutty phenomena' versus trying to squeeze them all into one category or the other, regardless of how unnatural the fit appears." (Beall, 2017, 13)

is white does not entail *grass is purple*, so Triv is too strong. Assuming that *snow is white and grass is green* do entail *snow is white*, Ni is too weak. In the middle, the entailment facts become more controversial, which makes things difficult for us. A paraconsistent logician will claim that classical logic is too strong because it contains $A, \neg A \vDash B$ and a classical logician will say that LP is too weak because it fails to contain the same theorem. The difficulties we have settling disputes over the answers to these questions don't change the fact that logic strength is something that a logic is supposed to get right, rather than something it is always good to have more of.

It is with some regret that I reach this conclusion. Logical strength would have made a nice theoretical virtue, because unlike some other characteristics, such as simplicity and elegance, it is relatively straightforward to see whether one logic is stronger than another, and in many cases of genuine dispute, the relative strengths of the two rival theories are not themselves in question. Since it is neither a vice nor a virtue, however, the clarity of logical strength rankings can make them a dangerous temptation: university professors sometimes complain that numerical student evaluations do not measure teaching effectiveness and worse, in the absence of some better measure, these false measures get used because administrators find them convenient, need a way to measure teaching effectiveness, and can find nothing better. If we can't find better ways to assess logical theories, the clear rankings of logics in terms of logical strength may tempt us to use them.

3 Scientific Strength

Logical strength is not the only relevant sense of "strength". Williamson identifies a different characteristic of theories which he calls *scientific strength.* This characteristic is harder to pin down (Williamson calls it a "looser" sense of strength) but the core idea is that scientifically stronger theories are more *informative and specific.* In Williamson's main example "The time is between 3.14 and 3.16" is scientifically stronger than "The time is between 4.00 and 12.00."[13] He suggests that logical strength entails scientific strength, but not vice versa:

> "If T is stronger than T* in the strict logical sense, then T is also stronger than T* in the looser scientific sense, but the converse fails. Both senses are applicable to logical theories. For instance, let PC be standard classical propositional logic, and IC be intuitionist propositional logic. Then every theorem of IC is a theorem of PC but not conversely, since $P \vee \neg P$ is a theorem of PC but not of IC; likewise for the corresponding consequence relations. Thus PC is stronger than IC in the strict logical sense, and so also in the looser scientific sense."

I think it very likely that scientific strength is a virtue. However, if Williamson's claim about the relationship between logical strength and scientific strength is

[13] (Williamson, 201X)

true, it threatens my conclusion from the previous section. For suppose scientific strength is a virtue. Then by my own standards, a theory which has more of it is ceteris paribus better. But then if L1 is logically stronger than L2, and logically stronger entails scientifically stronger, then L1 must be scientifically stronger than L2 as well. So the fact that L1 is logically stronger than L2 means that L2 is better than L2 after all. Logical strength would be a virtue.

But something about the idea that one contemporary logic is more informative and precise than another contemporary logic might strike us as odd. None of the logics we've mentioned so far say that the validity of modus ponens is between 3 and 7. Or even that the law of excluded middle is occasionally a logical truth. Modern logic is mathematical, and logics are formulated so that they are determinate, in the following sense: for any set of premises Γ and conclusion, A in the language on which the logic is defined, it says whether or not $\Gamma \vDash A$ for a particular Γ and A. A common way to define a logic, for example, is to first specify the language on which it is to be defined, then define a set of models U for that language, define the conditions under which a sentence will be true in a model, and finally use these things to give the conditions under which a set of premises is counted as entailing a conclusion, e.g. $\Gamma \vDash A$ if and only if there is no model $M \in U$ such that M makes every member of Γ true and does not make A true. Whether a model is a member of U is determinate, and (at least in the kinds of logics we've looked at so far in this paper) whether a model makes a sentence true is determine, so the model decides for each argument and argument form, either $\Gamma \vDash A$ or $\Gamma \nvDash A$. How could the logic be any *more* informative or specific than that?

Still, I do think there is a way in which scientific weakness tends to creep into logical theories, one to which weak logics are especially prone.

The Vagaries of Classical Recapture

The claims on which classical logic and subclassical logic differ are very general claims. For example, if you claim that modus ponens,

$$A \to B, A \vDash B$$

is valid, then *whatever* sentences form the antecedent and the consequent of the conditional, you still can't have a situation where the conditional and the antecedent are both true, but the consequent is not true. If modus ponens is valid it has to work *every* time, for every value of A and B. The same goes for logical truths. If you accept the law of excluded middle then you are committed to $A \vee \neg A$ being true for *all* sentences A.

This great generality means that non-classical logicians sometimes reject an entire logical principle because of a quite special case. For example, certain intuitionists in mathematics think that the law of excluded middle is not valid because it is not true for mathematical objects (such as far out members in infinite collections) which have not yet been constructed by us. But this exception to the law is quite specific—it concerns certain sorts of mathematical object—and an intuitionist need not think there are any problems with the law of excluded middle when it is restricted to say, sentences that make reference only to the first 10 natural numbers, or a finite collection of physical objects—they need have no quarrel with 3 is prime $\vee\neg$3 is prime, for example, or *every cup on my desk is dirty or not dirty.* These things are true. You can use them in arguments. They're just not *logically true*, according to the intuitionist.

Similarly, some dialetheists reject modus ponens because they hold that the Liar Paradox shows that some sentences can be both truth and false. This, they think, along with the LP interpretation of the connectives, generates counterexamples to the rule:

A	B	A	→	B,	A	B
T	T	T	T	T	T	T
T	F	T	F	F	T	F
T	B	T	B	B	T	B
F	T	F	T	T	F	T
F	F	F	T	F	F	F
F	B	F	T	B	F	B
B	T	B	T	T	B	T
B	F	B	B	F	B	F
B	B	B	B	B	B	B

But dialetheists can hold that lots of sentences are merely True, or merely False (not Both), and would maintain that you make no mistake when applying modus ponens to say '*if* $2 = S(1)$ *then* $1+1 = 2$ and $2 = S(1)$', or to *if snow is white then grass is green* and *snow is white*—in these cases the move won't take you from truth to non-truth—you just have to be careful not to endorse modus ponens as *logically* valid, since validity is something that applies everywhere—including in the dialetheic zones.

Given this, when responding to concerns about their logic being too weak, a non-classical logician might engage in a project that is sometimes called *classical recapture*. They say, essentially, "yes, my logic is a bit weak, but in practice you can use classical logic in all sorts of situations. It's just that where it goes beyond the real logic its truths aren't *logically* true." For example, here's Hartry Field:

> "...we ought to seriously consider restricting classical logic to deal with all these paradoxes. [...] I say 'restricting' rather than 'abandoning', because there is a wide range of circumstances in which classical logic works fine. Indeed, I take excluded middle to be *clearly* suspect only for certain sentences that have a kind of "inherent cirularity" because they contain predicates like 'true'; and most sentences with those predicates can be argued to satisfy excluded middle too. The idea is *not* that we need two logics, classical logic and a paracomplete logic (one without excluded middle.) On the contrary, the idea is that we can take the paracomplete logic

to be our single all-purpose logic. But we can recognise the truth of all instances of excluded middle in certain domains (e.g. those that don't contain 'true', 'true of', 'instantiates', or other suspect terms)." (Field, 2008, p.15)

At the extreme of this strategy there is my own work on logical nihilism.[14] What makes logical nihilism even remotely tenable is the idea that while no putative logical entailment holds for any language, no matter how odd, many familiar logical principles work just fine for familiar languages—you can accept lots of regular common-or-garden instances of modus ponens without accepting modus ponens as a logical law.

Still, this strategy for making weak logics more palatable intuitively threatens the informativeness and the precision of the theory—its scientific strength. In the next section I want to try to make this idea more precise, and argue against Williamson's connection between logical strength and scientific strength. I'm going to do this with the help of a simple—I think fairly easy to visualise—analogy.

Large Squares and Small Squares

Suppose you have a 10X10 grid of Large Squares, numbered 1–100. Large Squares have two states—black or grey. Each Large Square in the 10X10 grid is itself a 10X10 grid, subdivided into 100 Small Squares. Small squares too may be black or grey. If the 100 Small Squares that make up a Large Square are all black, then the Large Square is itself black. But if any of 100 Small Squares that make up a Large Square are grey, then the Large Square is grey.

A Large Square theory is a theory which gives information about the colour of the Large Squares. A Complete Large Square Theory will say, for each of the Large Squares 1–100, whether that square is black or grey. You might think of a Complete Large Square theory as 100 pairs of pairs of numbers with colours:

Theory A	
1	Black
2	Grey
3	Grey
4	Black
⋮	⋮

Theory B	
1	Black
2	Black
3	Grey
4	Black
⋮	⋮

Theory C	
1	Grey
2	Grey
3	Grey
4	Grey
⋮	⋮

When we restrict our attention to Large Squares, every Complete Large Square Theory is as strong every other Complete Large Square Theory. There are two states for each large square, and if a theory specifies exactly one of those states for each square, its work is done.

This is the situation with respect to rival logics defined on the same language. The job of the logical theory is to say for each argument form whether it is

[14] Russell (201X)

logically valid (black) or not (grey.) Triv, Classical logic, LP, K3, FDE, and Ni do this for every argument form, and hence they are on a par with respect to scientific strength.
A scientifically weaker theory would be one which was more vague or less informative than any of these familiar logics. Here are some candidates:

- T6: There are some valid arguments.
- T7. There are at least 8 valid arguments.
- T9: About half of all arguments are valid.
- T10: Somewhere between one and two thirds of all arguments are valid.
- T11: Arguments with conditionals in them are sometimes valid.

None of the logics we are seriously considering lack scientific strength in this way.

However, it might still seem as if there is still a sense in which classical logic is more informative. If you say that an argument form is valid, it follows that every instance of that form is a logically valid argument. For example, if I say that $A \to B, A \vDash B$ then it follows that $P \to Q, P \vDash Q$, where this is a instance of that general form. It also follows that these instances are valid:

Snow is white $\to$ grass is green	This literal is not true $\to$ snow is red.
Snow is white.	This literal is not true.
Grass is green.	Snow is red.

It is important to note that exactly the same goes for "not logically valid." If a theory says that the same argument form is not logically valid, it follows that *every* instance of that form is not logically valid.

However, there is a property of instances of argument forms that may be possessed by one instance and not another instance the same argument form, and is such that if every instance of an argument form has it, the resulting argument form is valid. Call that property *sub-validity*. Exactly what sub-validity is will vary with your view of consequence. For the Tarski of (Tarski, 1936) and (Etchemendy, 1999) it would have been having a true conclusion if you have all true premises (where the *if* is an ordinary material conditional.) But we might like to think of it more substantially as being necessary truth-perservation, or (my preferred option) being truth-preserving in virtue of meaning.

For example, the argument form $A \to B, B \vDash A$ is counted logically valid by none of our logics except Triv. That means that they (correctly) count both of the following as not being of logically valid form:

Snow is white $\to$ grass is green	Snow is coloured$\to$ snow is white.
Grass is green.	Snow is white.
Snow is white.	Show is coloured.

Even though the argument on the right is not an instance of a valid argument form, you wouldn't go wrong if you used this particular argument as a guide to the truth. The premises *do* guarantee the truth of the conclusion, moreover, if every instance of affirming the consequent had this property, there would be no counterexamples—the argument form would be valid.

So here is the sense in which logically stronger logics do seem to be scientifically stronger than logically weaker logics: they tell us more about which argument instances are subvalid. In telling us that modus ponens is logically valid, classical logic tells us that all argument instances of modus ponens are subvalid. But in telling us that modus ponens is not valid, LP tells us only that some argument instances of modus ponens are not subvalid.

Analogously, a Large Square Theory which tells us that squares 1-50 are black doesn't just tell us about the colour of Large Squares, it also functions as a theory of Small Squares. A theory of Small Squares tells us which colour the Small Squares are. A Complete Small Square Theory tells us the colour of every Small Square. One might think of a Complete Small Square theory as 1000 pairs of pairs of numbers with colours (the first number represents the Large Square in which the Small Square is embedded, the second its position in that Large Square).

Theory A									...
1	1	Black	2	1	Grey	3	1	Black	...
1	2	Grey	2	2	Grey	3	2	Grey	...
1	3	Grey	2	3	Black	3	3	Black	...
1	4	Black	2	4	Grey	3	4	Black	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

The only Large Square Theory which is Small Square Complete is the one which says that every Large Square is black, because that entails that all 1000 Small Squares are black too. But in general, the more Large Squares a Large Square theory says are black, the more informative it will be about the colour of Small Squares. In particular, if a theory says that Large Squares 1-50 are black, it will tell us the colour of 500 particular Small Squares (the ones embedded in Large Squares 1-50). Whereas if the theory says only that Large Squares 1-40 are black but 41-50 are grey, then it will tell us the colour of just 400 Small Squares, along with the information that at least one Small Square in each of the Large Squares between 41 and 50 is grey.

Hopefully these examples have made the analogy clear enough that I can use it to make some points about the relative logical and scientific strengths of classical and subclassical logics.

1. when we restrict our attention to argument forms and logical validity, all of the logics in the set {Triv, Cl, K3, LP, FDE, Ni} are on a par with respect to scientific strength—despite being unequal in logical strength—because they all give a definitive verdict on whether each argument form is logically valid or invalid.

2. When we expand our attention to include argument instances, still, all of the logics are on a par with respect to scientific strength because they all give a definitive verdict on whether each argument instance is of a logically valid or invalid form.

3. But, when we expand our attention yet further to include the property of subvalidity—a property such that, if every instance of an argument form has it the argument form is logically valid—then logically stronger logics really do tell us more than logically weaker logics. They are more informative—scientifically stronger—because saying that an argument form is valid tells us that *all* of its instances are subvalid, whereas saying that it is invalid merely tells us that at least one instance is subinvalid.

4. One might be tempted to defend the scientific strength of sub-classical logics on the grounds that we shouldn't expect them to tell us about things other than what they are a theory of. LP is a theory of logical validity, not subvalidity (this line would run) and so its scientific strength with respect to facts about subvalidity is irrelevant.

5. But this seems forced. Surely the reason we are interested in logical form at all is that we want to know which particular arguments are good. We've tended to study logical validity, but the weaker our logic the more important subvalidity starts to seem. If modus ponens isn't logically valid but some instances are subvalid, we would like to know which ones.

6. What we should note is that this difference in scientific strength is not inevitable. We can make a logical theory scientifically stronger without making the logic any stronger. To do that we need to make it a complete theory of subvalidity. (We need to make it Small Square Complete.)

7. As already mentioned, the only logic which is a complete theory of subvalidity all on its own is Triv—that's because it says that every argument instance is subvalid.

8. But any weaker logic can be extended to a complete theory of subvalidity—just as any complete theory of Large Squares could be extended to a complete theory of Small Squares. We just need to add the missing values, i.e. for each argument instance, say whether it is subvalid or not.

9. Some non-classical logicians have made a start on some of this already, with the project of classical recapture. Informativeness—essentially small square completeness—will be very important to the value of such attempts to increase scientific strength.

10. What the above suggests is that if scientific strength of this kind is really important, then i) it is required of classical logic as well as of weaker logics and ii) it must be done so that for each argument instance, the question of subvalidity is settled. The resulting theory should answer all

argument instance/subvalidity questions. It will not be enough to point to the mere possibility that, e.g. modus ponens might be ok when doing number theory, even if it is unacceptable when thinking about the semantic paradoxes. Of course, if scientific strength with respect to argument instances and subvalidity is not important after all, then we have yet to see an argument that classical logic is scientifically stronger than standard subclassical logics.

11. Finally, it isn't true that if a logic L1 is logically stronger than a logic L2, then L1 is also scientifically stronger than L2. Classical logic is logically stronger than LP, but they are equal in scientific strength with respect to Large Squares, and they *could*—through supplementing the theory without changing the logic—both be equal in strength with respect to Small Squares as well.

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